Dynamics of skyrmions in chiral ferromagnets

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A ferromagnetic film

The magnetisation vector $\mathbf{M} = \mathbf{M}(x, y, t)$

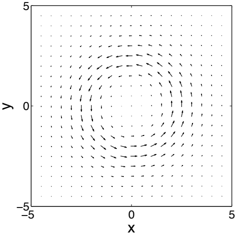
$\mathbf{M}^2(x, y, t) = M_s^2$, we typically normalise $\mathbf{m} = \mathbf{M}/M_s$, thus $\mathbf{m}^2 = 1$.

The skyrmion number

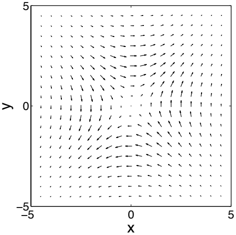
is a topological invariant and it counts the number of times that the magnetisation configuration $\mathbf{m}$ covers the sphere $\mathbf{m}^2 = 1$:

$$Q = \frac{1}{4\pi} \int q \, d^2x, \quad q = \frac{1}{2} \epsilon_{\mu\nu} \mathbf{m} \cdot (\partial_\nu \mathbf{m} \times \partial_\mu \mathbf{m})$$ topological density

Skyrmion ($Q = 1$)



Antiskyrmion ($Q = -1$)



Antisymmetric exchange interaction: Dzyaloshinskii-Moriya (DM) materials

A typical and minimal energy functional for $\mathbf{m} = (m_1, m_2, m_3)$ is

$$E = E_{\mathrm{ex}} + E_{\mathrm{a}} + E_{\mathrm{DM}}.$$

- The usual symmetric exchange energy

$$E_{\mathrm{ex}} = \frac{1}{2} \int \partial_\mu \mathbf{m} \cdot \partial_\mu \mathbf{m} \, d^2x, \qquad \mu = 1, 2.$$

- An easy-axis anisotropy energy (with constant $\kappa > 0$)

$$E_{\mathrm{a}} = \frac{\kappa}{2} \int (m_1^2 + m_2^2) \, d^2x.$$

- An exchange of the Dzyaloshinskii-Moriya type ($\lambda = \pm 1$)

$$E_{\mathrm{DM}} = \lambda \int \mathbf{m} \cdot (\nabla \times \mathbf{m}) \, d^2x.$$

The Landau-Lifshitz (LL) equation

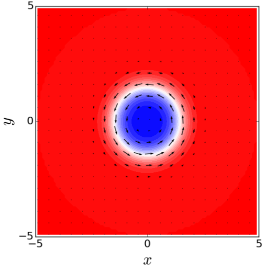
The conservative (Hamiltonian) LL equation associated with the energy is

$$\frac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times \mathbf{f}, \qquad \mathbf{m}^2 = 1$$

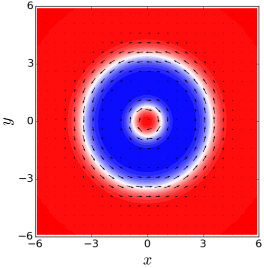
$$\mathbf{f} \equiv -\frac{\delta E}{\delta \mathbf{m}} = \Delta \mathbf{m} + \kappa m_3 \hat{\mathbf{e}}_3 - 2\lambda \nabla \times \mathbf{m}.$$

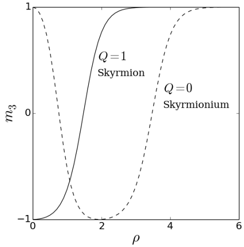
Static solutions in a film: $\mathbf{m} \times \mathbf{f} = 0$

Skyrmion ($Q = 1$)



Skyrmionium ($Q = 0$)





Stable excited states for $\kappa \geq (\pi^2/4)\lambda^2$

[A. N. Bogdanov and A. Hubert (1999)]

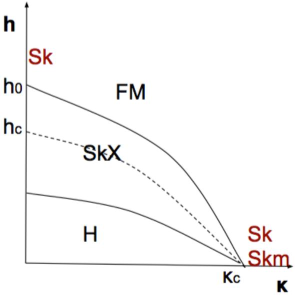
Existence: [Melcher, Proc. R. Soc. A 2014]

Skyrmionium-type configs (no DM):

[Moutafis, et al, PRB (2007)]

[Finazzi, et al, PRL (2013)]

Phase diagram (sketch)



H: helix, **FM**: ferromagnetic state, **SkX**: skyrmion lattice (ground states)
Sk: skyrmion, **Skm**: skyrmionium (excited states)

$$h_c = \pi^2/16, \quad h_0 \approx 0.8, \qquad \kappa_c = \pi^2/4.$$

Dynamics of skyrmions

Fundamental relation for evolution of topological density [Papanicolaou, Tomaras, 1991]:

$$\dot{q} = -\epsilon_{\mu\nu}\partial_\mu(\mathbf{f}\cdot\partial_\nu\mathbf{m}) = \epsilon_{\mu\nu}\,\partial_\mu\partial_\lambda\sigma_{\nu\lambda}, \quad \mu,\nu,\lambda = 1,2$$

where $\mathbf{f}\cdot\partial_\mu\mathbf{m} = -\partial_\nu\sigma_{\mu\nu}$.

The tensor $\sigma_{\mu\nu}$ has components

$$\begin{aligned}
\sigma_{11} =& \frac{1}{2}\left(\partial_2\mathbf{m}\cdot\partial_2\mathbf{m} - \partial_1\mathbf{m}\cdot\partial_1\mathbf{m}\right) + \frac{\kappa}{2}(m_1^2+m_2^2) + \lambda(m_1\partial_2 m_3 - m_3\partial_2 m_1 \\
\sigma_{12} =& -\partial_1\mathbf{m}\cdot\partial_2\mathbf{m} + \lambda(m_3\partial_1 m_1 - m_1\partial_1 m_3) \\
\sigma_{21} =& -\partial_1\mathbf{m}\cdot\partial_2\mathbf{m} + \lambda(m_2\partial_2 m_3 - m_3\partial_2 m_2) \\
\sigma_{22} =& \frac{1}{2}\left(\partial_1\mathbf{m}\cdot\partial_1\mathbf{m} - \partial_2\mathbf{m}\cdot\partial_2\mathbf{m}\right) + \frac{\kappa}{2}(m_1^2+m_2^2) + \lambda(m_3\partial_1 m_2 - m_2\partial_1 m_3
\end{aligned}$$

Dynamics of skyrmions: I_μ

Define the moments of topological density q:

$$I_\mu = \int x_\mu q \, d^2 x \qquad \mu = 1, 2.$$

Prove that they are conserved $\dot{I}_\mu = 0$ (by application of fundamental relation in previous page).

A rigid translation of spatial coordinates by a constant vector

$$x_\mu \to x_\mu + c_\mu \quad \Rightarrow \quad I_\mu \to I_\mu + 4\pi Q \, c_\mu$$

reveals difference in dynamics between topological ($Q \neq 0$) and non-topological ($Q = 0$) magnetic solitons.

- For $Q \neq 0$, the (I_1, I_2) gives position of skyrmion (it is fixed).
- For $Q = 0$, skyrmions may propagate freely (solitary waves).

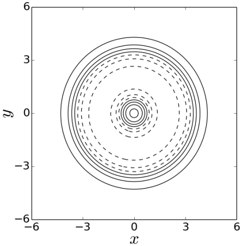
$Q = 0$ skyrmionium as a traveling wave

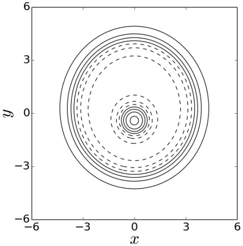
Assume propagating skyrmionium with velocity v (solitary wave). We make the traveling wave ansatz $\mathbf{m} = \mathbf{m}(x - vt, y)$ and this satisfies

$$v \frac{\partial \mathbf{m}}{\partial x} = \mathbf{m} \times \mathbf{f}.$$

We find numerically traveling solutions for $0 \leq v < v_c \approx 0.102$

$v = 0$ $\qquad\qquad$ $v = 0.07$



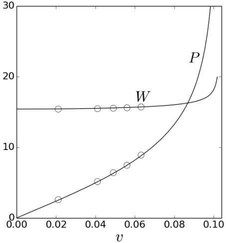


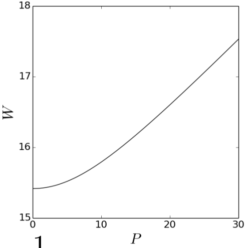
m_3 contour plots (solid lines: $m_3 > 0$, dashed lines: $m_3 < 0$)

Energy – Momentum relation

The linear momentum $\mathbf{P} = (P_1, P_2)$ is defined by

$$P_\mu = \epsilon_{\mu\nu} I_\nu \quad \text{or} \quad \mathbf{P} = (I_2, -I_1).$$





$$(P_1 =)\ P = mv, \qquad E = E_0 + \frac{1}{2} m v^2, \qquad v \ll v_c$$

We may associate a mass (m) to the skyrmionium

At low momenta $E = E_0 + \frac{P^2}{2m}$ (Newtonian)

At high momenta $E \approx v_c P$ (relativistic).

Force and acceleration on a $Q = 0$ skyrmionium

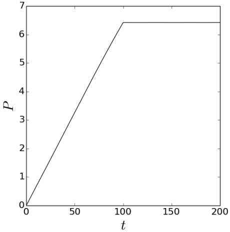
Apply an external non-homogeneous magnetic field, e.g.,

$$\mathbf{h} = (0, 0, h), \qquad h = g\,x.$$

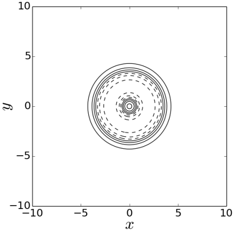
The force changes the linear momentum

$$\dot{P}_x = -\int \partial_x h (1 - m_3)\, d^2x, \quad \dot{P}_y = 0.$$

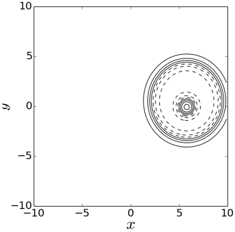
Force for $t \leq 100$



$t = 0$



$t = 160$



Skyrmion dynamics for $Q = 0$

When forced it accelerates. Propagates freely in the absence of force.

Force on $Q = 1$ skyrmions

Apply a magnetic field gradient

$$\mathbf{h} = (0, 0, h), \qquad h = gy.$$

Skew deflection of magnetic bubbles in field-gradient

[Malozemoff, Slonczewski, "Magnetic Domain Walls in Bubble Materials", 1979]

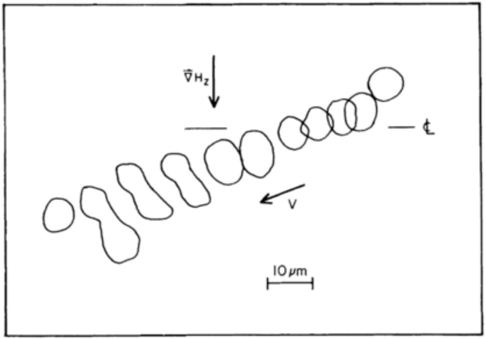


Fig. 13.2. Initial and final normal photographs and nine intermediate superimposed high-speed photographs of a hard bubble at the end of each of a sequence of nine gradient pulses of length 2 μsec and strength $H_g = |r\nabla H_z| = 4.5$ Oe oriented as indicated in a EuGaYIG film. The overall direction of the bubble motion illustrates the skew deflection of hard bubbles and the elliptical transient shape suggests a bunching effect. The horizontal lines indicate the center line of the gradient (after Patterson *et al.*[357]).

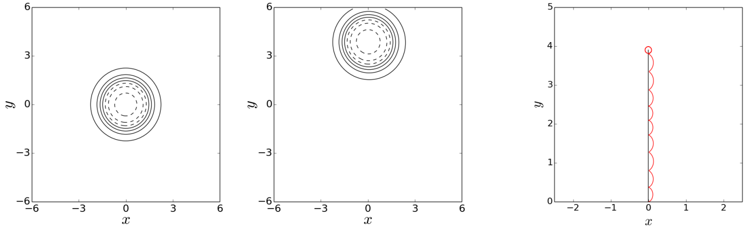
Hall motion of $Q = 1$ skyrmion

We follow the skyrmion guiding center $\mathbf{R} = (R_1, R_2)$:

$$R_\mu = \frac{I_\mu}{4\pi Q} = \frac{1}{4\pi Q} \int x_\mu q \, d^2x.$$

The evolution equations are calculated as

$$\dot{R}_x = 0, \quad \dot{R}_y = -\frac{1}{4\pi Q} \int \partial_x h \, (1 - m_3) \, d^2x.$$



Skyrmion dynamics for $Q \neq 0$

When forced, propagates with constant velocity.
It is spontaneously pinned in the absence of force.

Skyrmion dynamics under spin-transfer torque (and damping)

The equation of motion is

$$(\partial_t + u\,\partial_1)\mathbf{m} = -\mathbf{m} \times \mathbf{f} + \mathbf{m} \times (\alpha\partial_t + \beta u\,\partial_1)\,\mathbf{m}$$

where β, u are the spin torque parameters and α the damping.
For $\alpha = \beta$ we get

$$(\partial_t + u\,\partial_1)\mathbf{m} = -\mathbf{m} \times \mathbf{f} + \alpha\mathbf{m} \times (\partial_t + u\,\partial_1)\,\mathbf{m}$$

that is, the LLG where the time derivative is $\partial_t + u\,\partial_1$.
Consider the traveling wave $\mathbf{m}(x_1, x_2, t) = \mathbf{m}_0(x_1 - ut, x_2)$, for which $\partial_t\mathbf{m} = -u\partial_1\mathbf{m}$. The equation reduces to the static LL:

$$\mathbf{m} \times \mathbf{f} = 0.$$

Skyrmion dynamics for $\alpha = \beta$

If we apply spin torque to a static solution (skyrmion) of the LL then this is set in motion with velocity $(u, 0)$.

Skyrmionium dynamics under spin-transfer torque

For $\alpha \neq \beta$, assume traveling configuration with velocity $(v, 0)$:

$$(u - v)\partial_1 \mathbf{m} = -\mathbf{m} \times \mathbf{f} + (\beta u - \alpha v)\, \mathbf{m} \times \partial_1 \mathbf{m}$$

We set $v = \beta u / \alpha$ and have

$$(v - u)\partial_1 \mathbf{m} = -\mathbf{m} \times \mathbf{f}$$

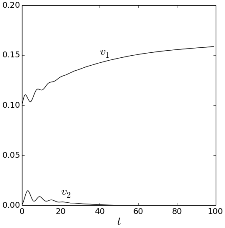
that is, the LL for a traveling wave – already solved for a skyrmionium.

Traveling wave

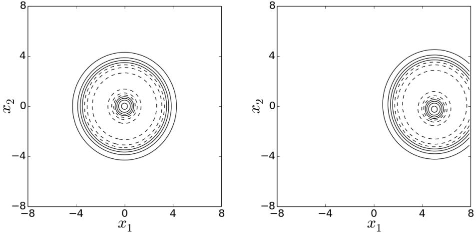
A skyrmionium traveling with velocity v under spin torque is identical to a skyrmionium traveling with velocity $v - u$ in the LL (without spin torque).

Simulation of skyrmionium dynamics

Spin torque is applied to a static skyrmionium at time $t = 0$. Velocity components $(v_1(t), v_2(t))$.
Parameter values $\alpha = 0.06, \quad u = 0.1, \quad \beta = 0.1.$



$(v_1, v_2) \to (0.167, 0)$



Skyrmionium at time $t = 0$ and $t = 40$.

Prescription

Consider a static skyrmionium

- Apply spin current for long enough time.
- The skyrmionium is set in motion with velocity $v = \beta u/\alpha$.
- Switch-off the current.
- The skyrmionium continues free motion with velocity $v - u$.

Free motion

For $\alpha \neq \beta$ the configuration is deformed by the spin current and is set in motion. After switching off the current it continues propagating but its velocity is reduced by u.

Concluding remarks

- The Dzyaloshinskii-Moriya interaction in ferromagnetic materials supports both topological and non-topological magnetic configurations.
- A topological $Q \neq 0$ skyrmion is pinned in a ferromagnetic film. It moves perpendicular to an applied force. The dynamics is analogous to the motion of an electron in a perpendicular magnetic field.
- A non-topological $Q = 0$ skyrmionium may move freely as a solitary wave. It responds as a Newtonian particle to forces.
- A skyrmionium can be set into free motion using external forces or a spin current.